Geometric Modeling

Alexander Pasko, Evgenii Maltsev, Dmitry Popov

Bird’s eye view of the course

Lecture topics:

I. Basics of shape modeling

II. Curves and surfaces

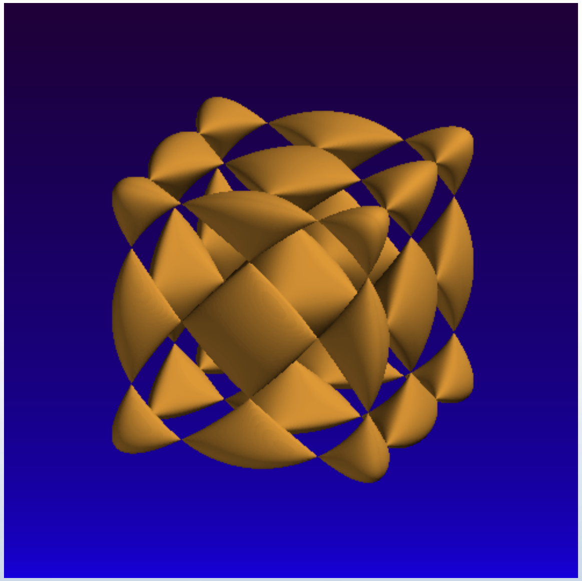
III. Transformations

IV. Solid modeling

V. Procedural modeling

VI. Applications

Geometric Modeling

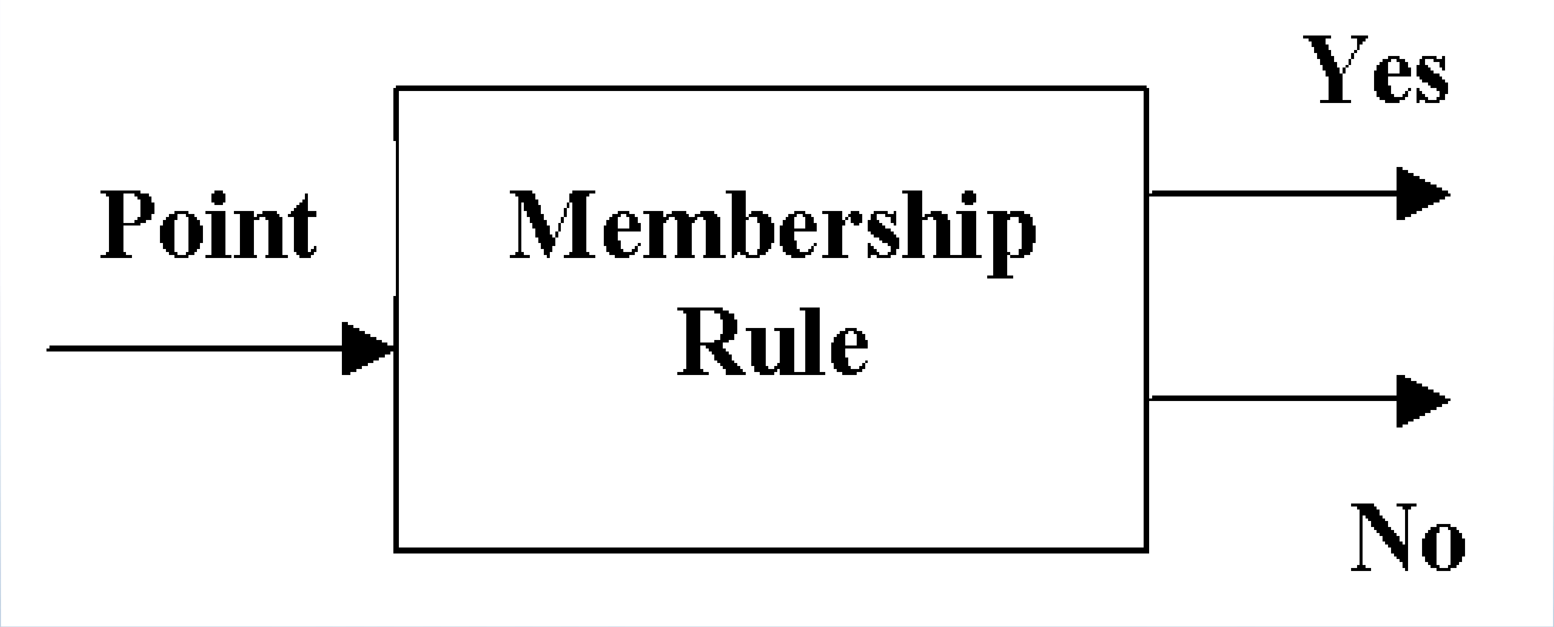


Implicit Curves and Surfaces

Contents

- *Point membership rule*
- *Implicit curves*
- *Algebraic surfaces*
- *Skeletal implicit surfaces*
- *Convolution surfaces*

Point Membership Rule



Implicit curves and surfaces

Implicit Curves and Areas

A set of points on a 2D plane with

$$f(x,y) = 0$$

is called an implicit curve

A 2D area (piece of plane, planar area, 2D solid) is defined as

$$f(x, y) \geq 0$$

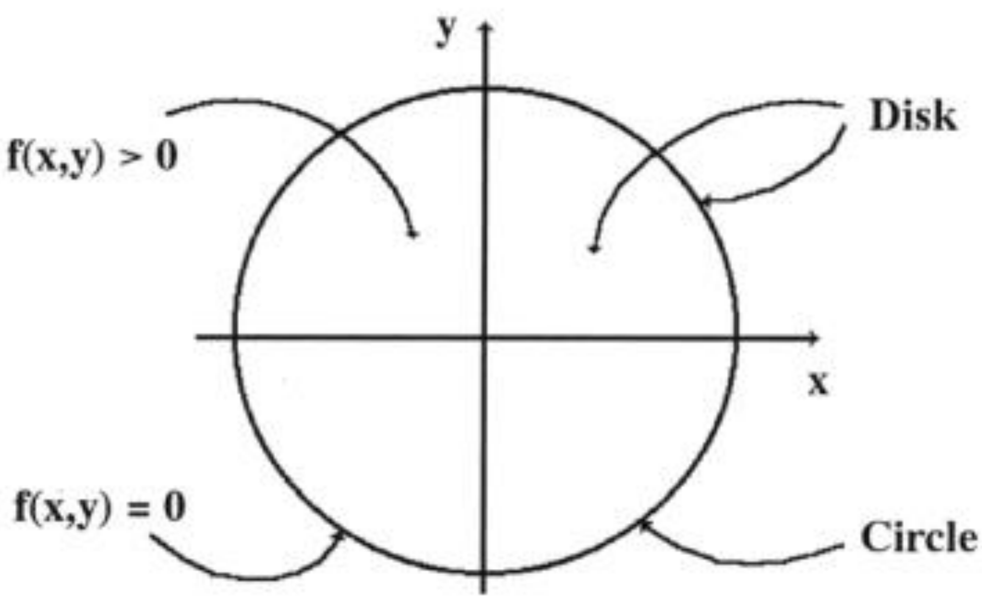
with the implicit curve as its boundary.

Circle and Disk

$$f(x,y) = R^2 - x^2 - y^2$$

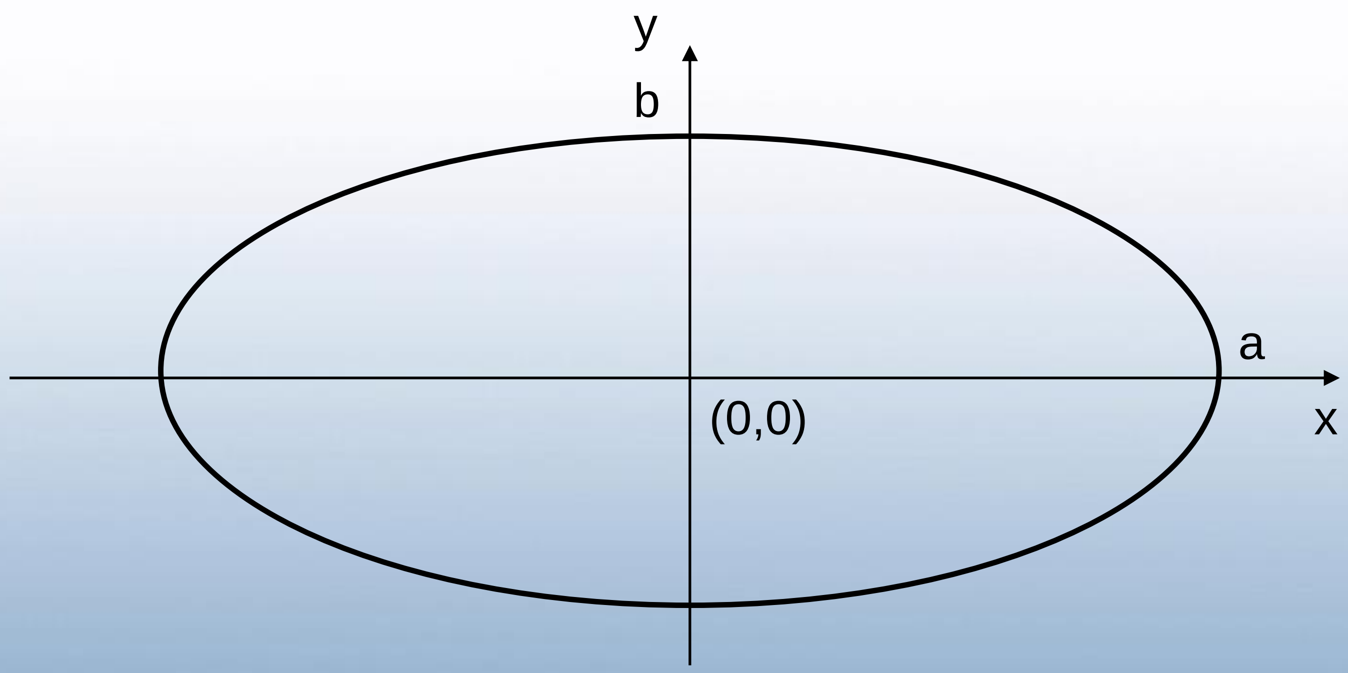
Disk (k=2) $f(x,y) \geq 0$

Circle (k=1) $f(x,y) = 0$



Ellipse

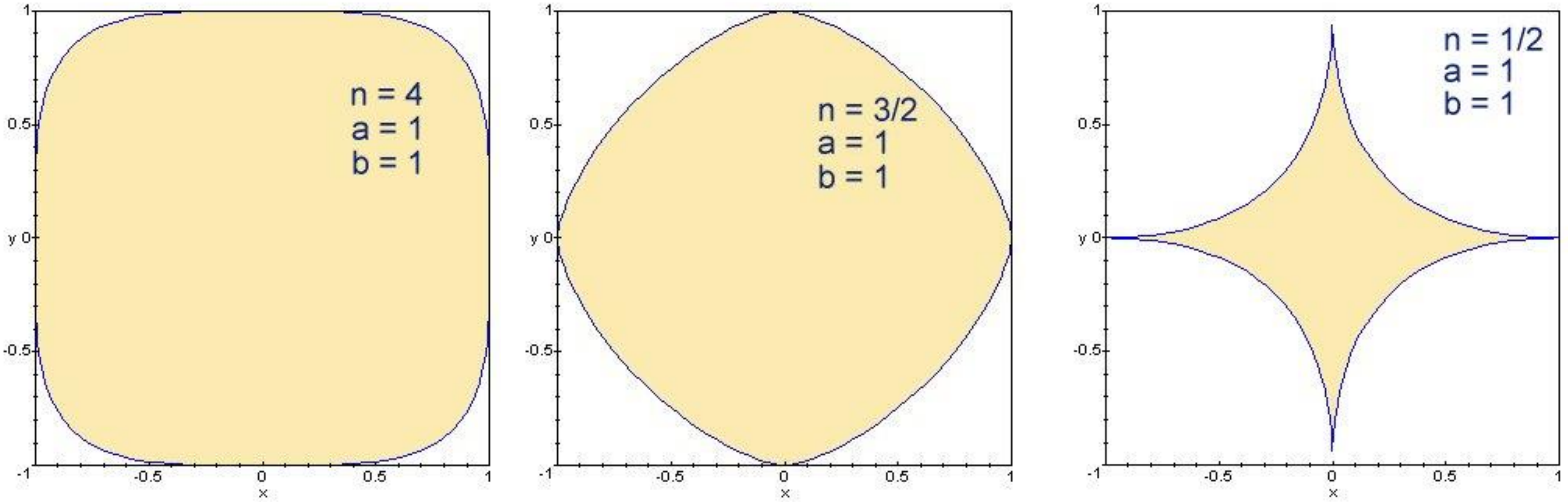
$$F(x,y) = 1 - (x/a)^2 - (y/b)^2$$



$$F(x,y) = 1 - ((x-x_0)/\, a)^2 - ((y-y_0)/\, b)^2 \quad ?$$

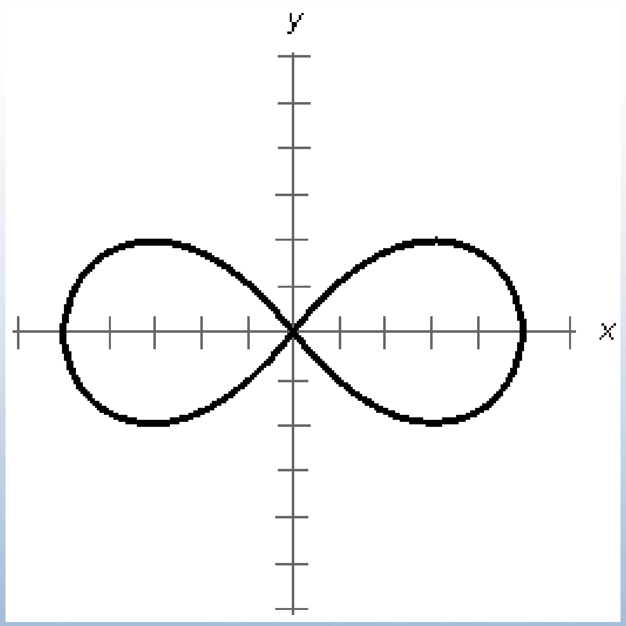
Superellipse

$$F(x,y) = 1 - (x/a)^n - (y/b)^n$$

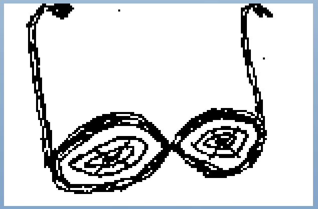


Lemniscate of Bernoulli

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$



The lemniscate was first described in 1694 by Jakob Bernoulli as a modification of an ellipse The curve has become a symbol of infinity and is widely used in math.



“Implicit” Surfaces

A set of points with

$$f(x,y,z) = 0$$

is called an “implicit” surface

Is the term “implicit” correct?

Explicit or "Implicit"?

Two ways to define z as a function of x,y:

1) Explicit form

$$z = \phi(x,y)$$

2) Implicit form

$$f(x,y,z) = 0$$

Both forms are **not enough** to define a sphere, **only** a hemi-sphere.

Iso-valued surface

$$\xi = f(x,y,z)$$

is an explicit function of three variables

$$\xi = 0 \quad \text{or} \quad f(x,y,z) = 0$$

is an iso-valued surface(*isosurface*) of a function of three variables

(or an “implicit” surface as the historical accident)

"Implicit" Surface - OK

Implicit Function - NO

3D Solids

A 3D solid is defined as

$$f(x, y, z) \geq 0$$

Isosurface f=0 or “implicit” surface is the boundary of this solid.

Classes of implicit surfaces

- Algebraic surfaces
- Skeletal implicit surfaces
- Convolution surfaces

Algebraic Surfaces

Surfaces with polynomial $f(x,y,z)$

- Quadratic
  - Sphere
  - Ellipsoid
  - Cylinder
  - Cone
  - Paraboloid, …
- Torus
- Superellipsoids

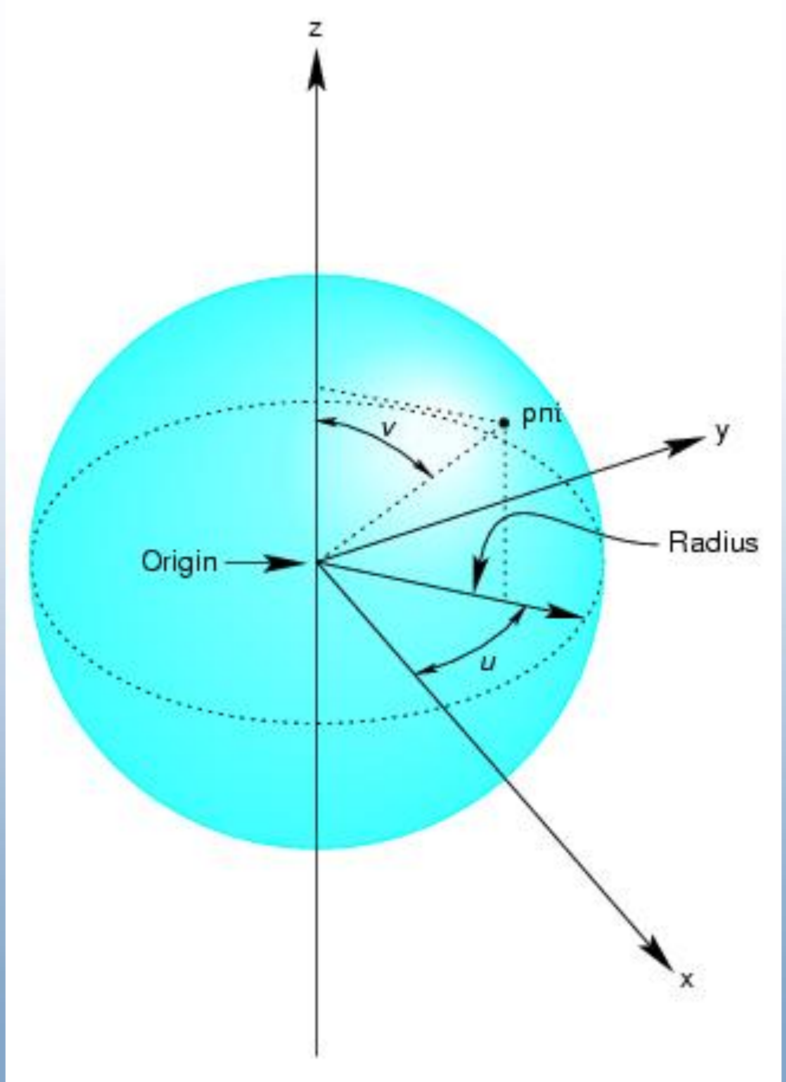
Sphere and Solid Ball

Sphere surface:

$$R^2 - x^2 - y^2 - z^2 = 0$$

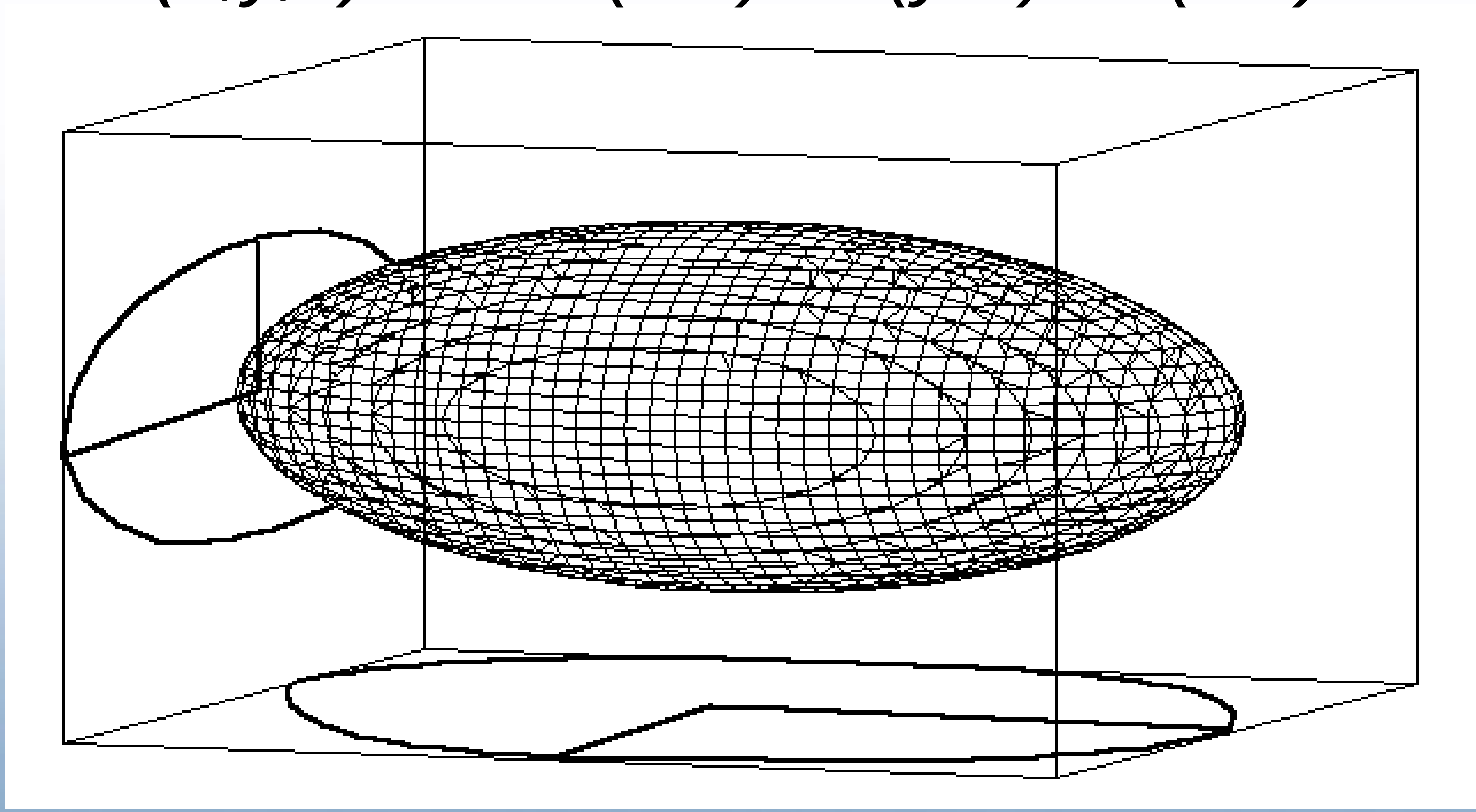
Solid ball:

$$R^2 - x^2 - y^2 - z^2 \geq 0$$



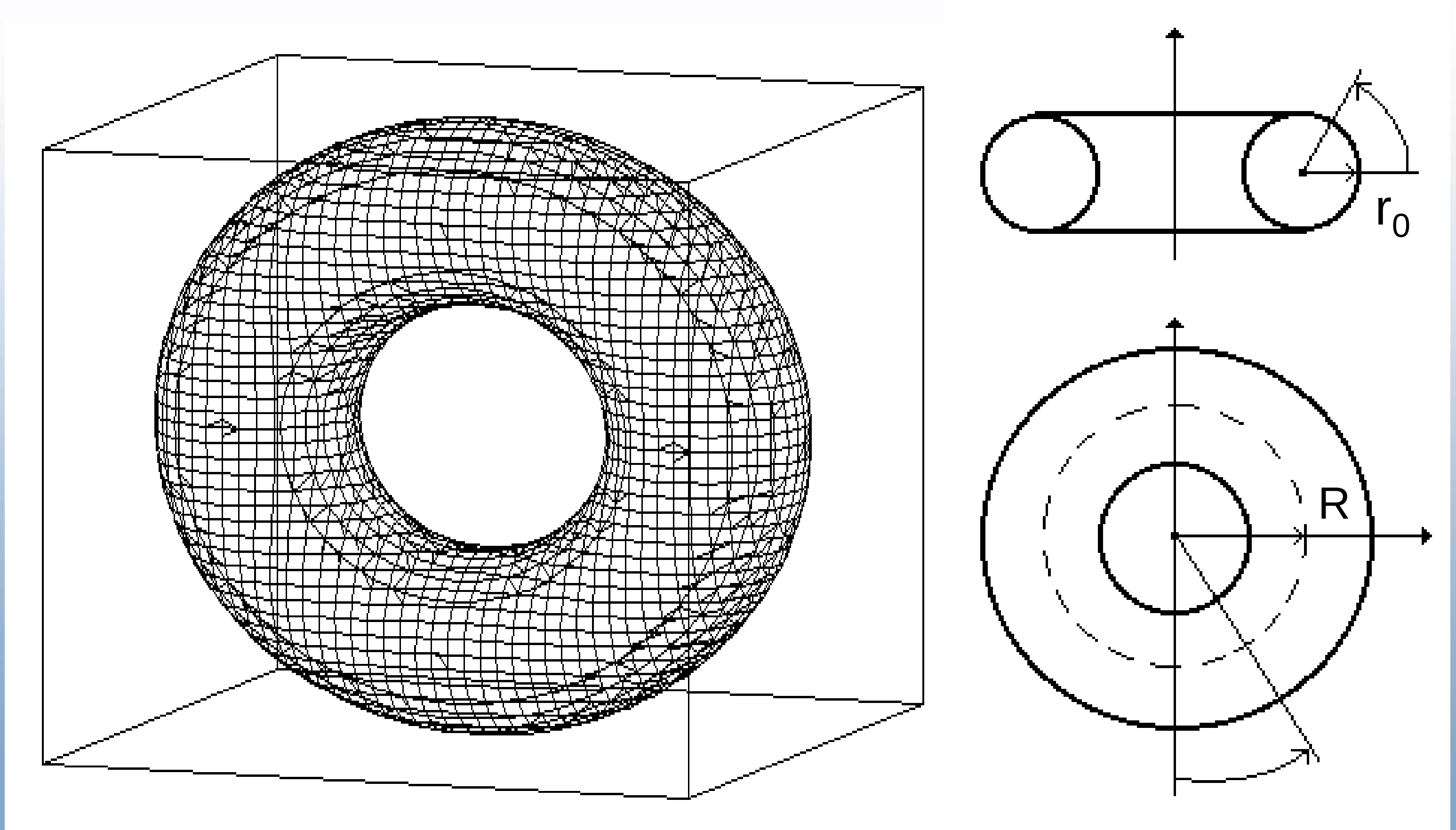
Ellipsoid

$$F(x,y,z) = 1 - (x/a)^2 - (y/b)^2 - (z/c)^2$$



Torus

$$F(x,y,z) = r_0^2 - x^2 - y^2 - z^2 - R^2 + 2R\sqrt{x^2+y^2}$$



Superellipsoids

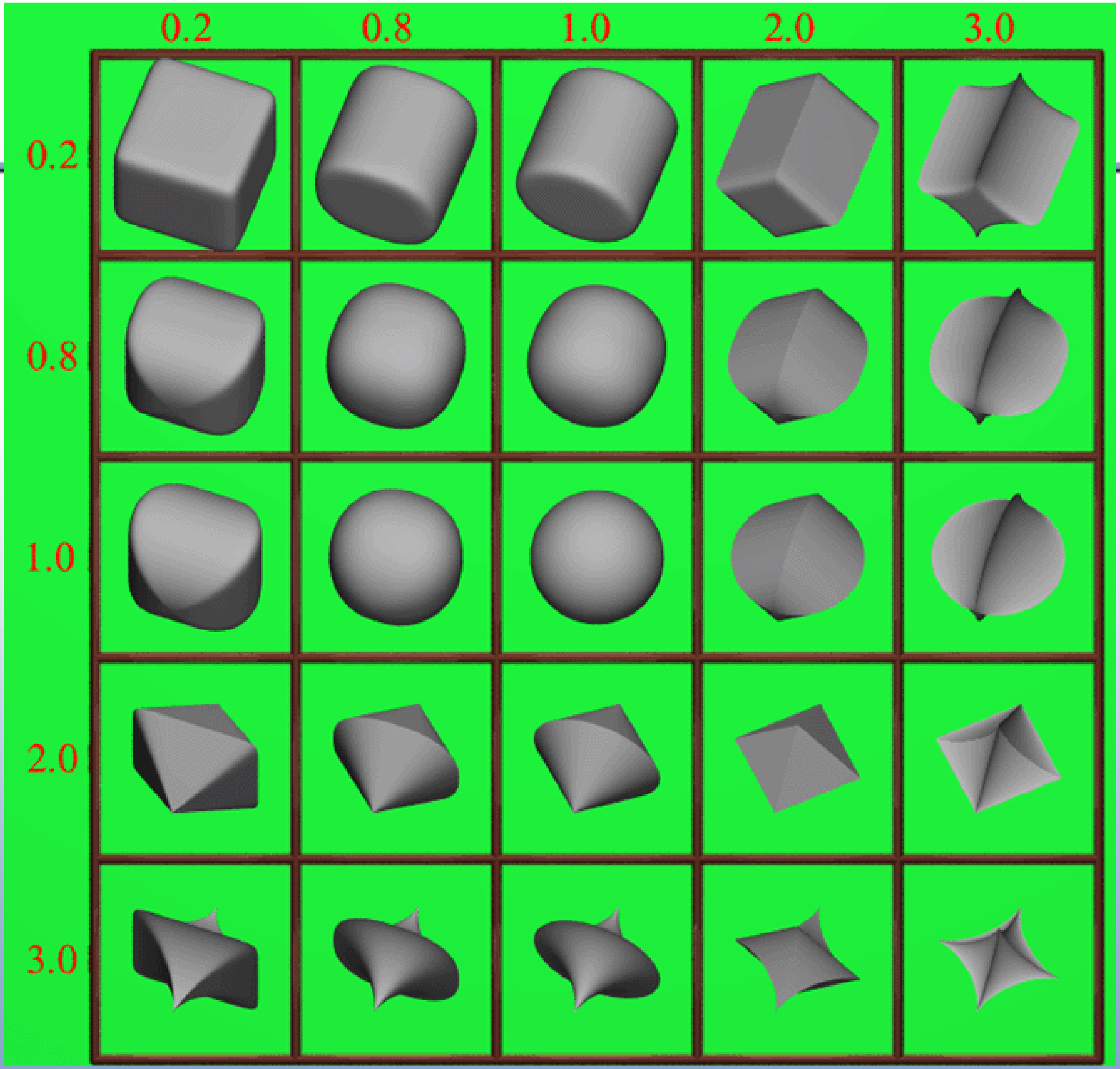
$$f(x,y,z) = 1 - \left[\left(\frac{x}{r_x} \right)^{\frac{2}{s_2}} + \left(\frac{y}{r_y} \right)^{\frac{2}{s_2}} \right]^{\frac{s_2}{s_1}} - \left(\frac{z}{r_z} \right)^{\frac{2}{s_1}}$$

s_1 and s_2 – shape control parameters

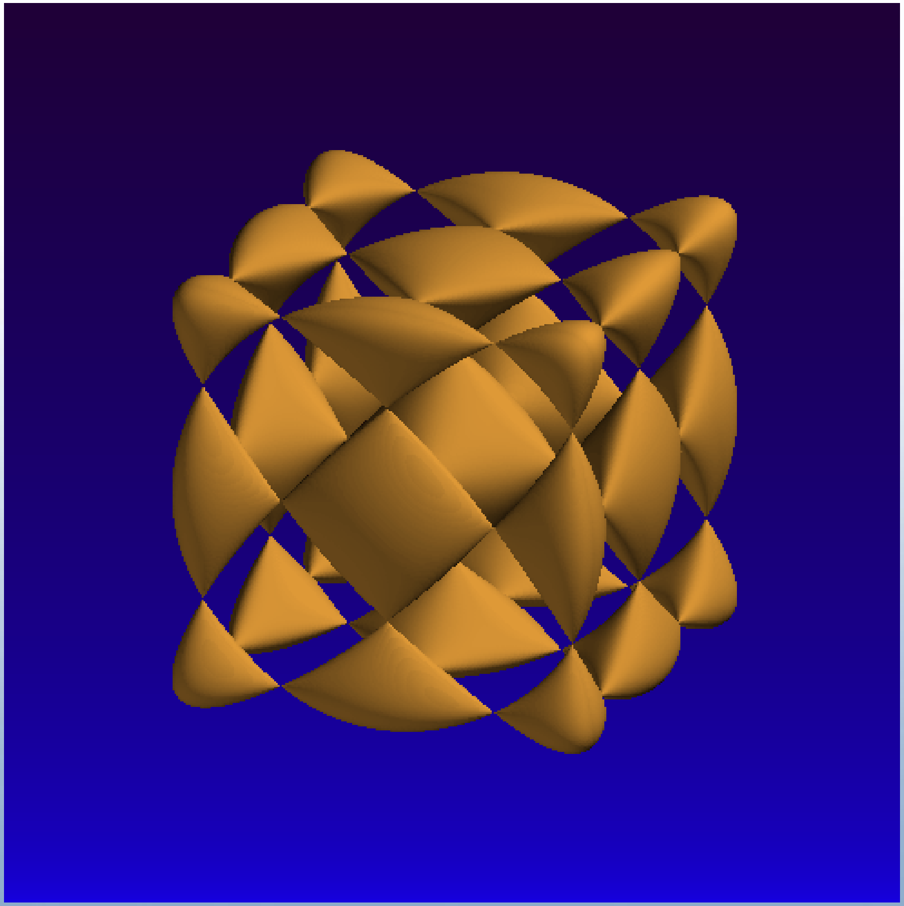
Superellipsoids

s_2

s_1



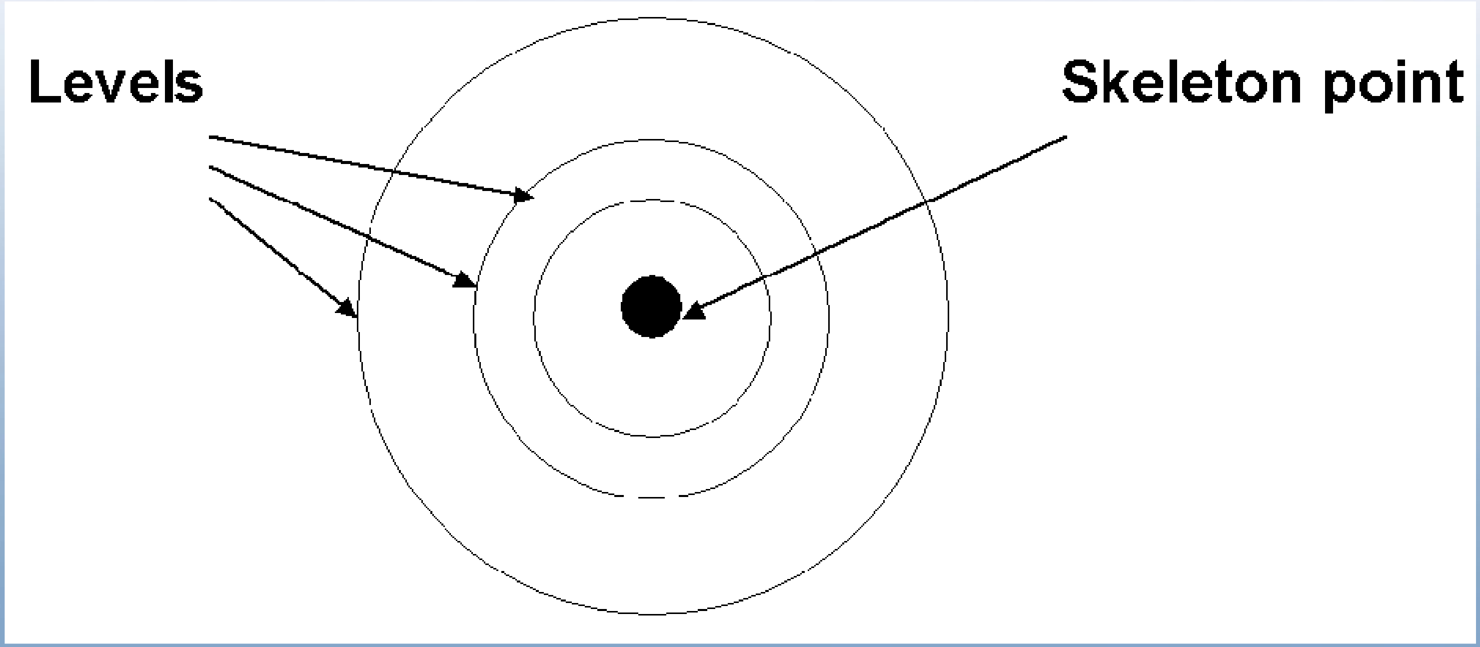
Chebyshev Polynomial



Skeletal Implicit Surfaces

Blinn [1982]:

modeling isosurfaces as a side effect of visualizing electron density fields

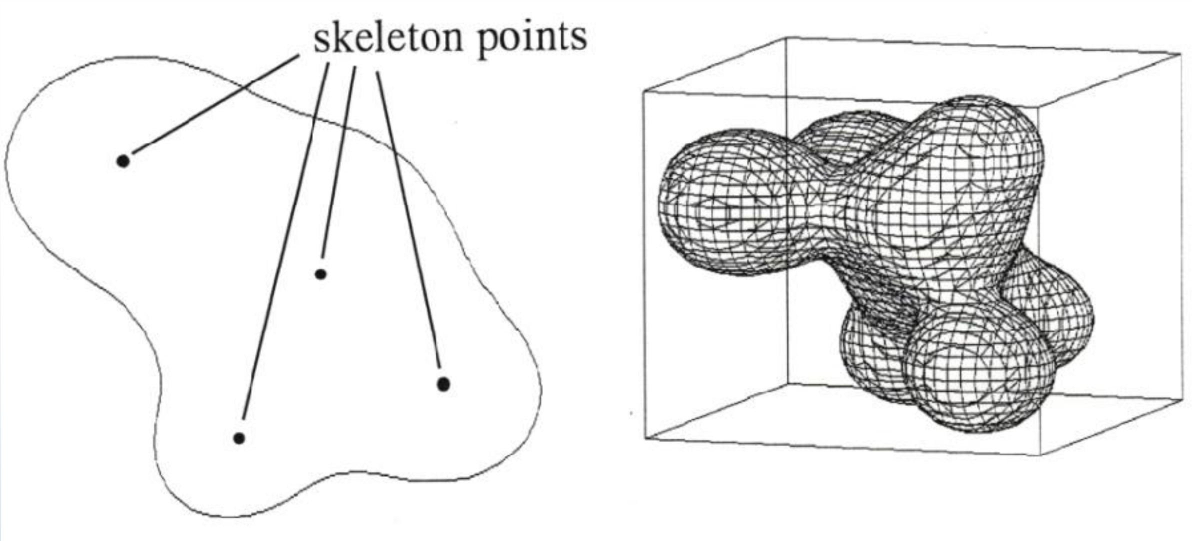


Skeletal model elements:

- *Skeleton* (points, lines and others);
- *Scalar field* with an individual skeleton element as a source;
- *Global field* as an *algebraic sum* of individual fields;
- *Level* (or *threshold*) of the field value defining the isosurface of interest.

Skeletal Implicit Surfaces

2D isoline　　　　3D isosurface



Skeletal Surface Definition

$$F(P) - T = 0$$

with $$F(P) = \sum_{i=1} c_i F_i(r_i)$$

N is the number of skeletal elements,

F_i is the individual scalar field,
(*blending function*) of the *i*-th element,

r_i is the distance from *P* to the *i*-th element,

T is the *threshold* (or *level value*).

Blobby models, metaballs, soft objects

Blobby Model

Blinn [1982]:

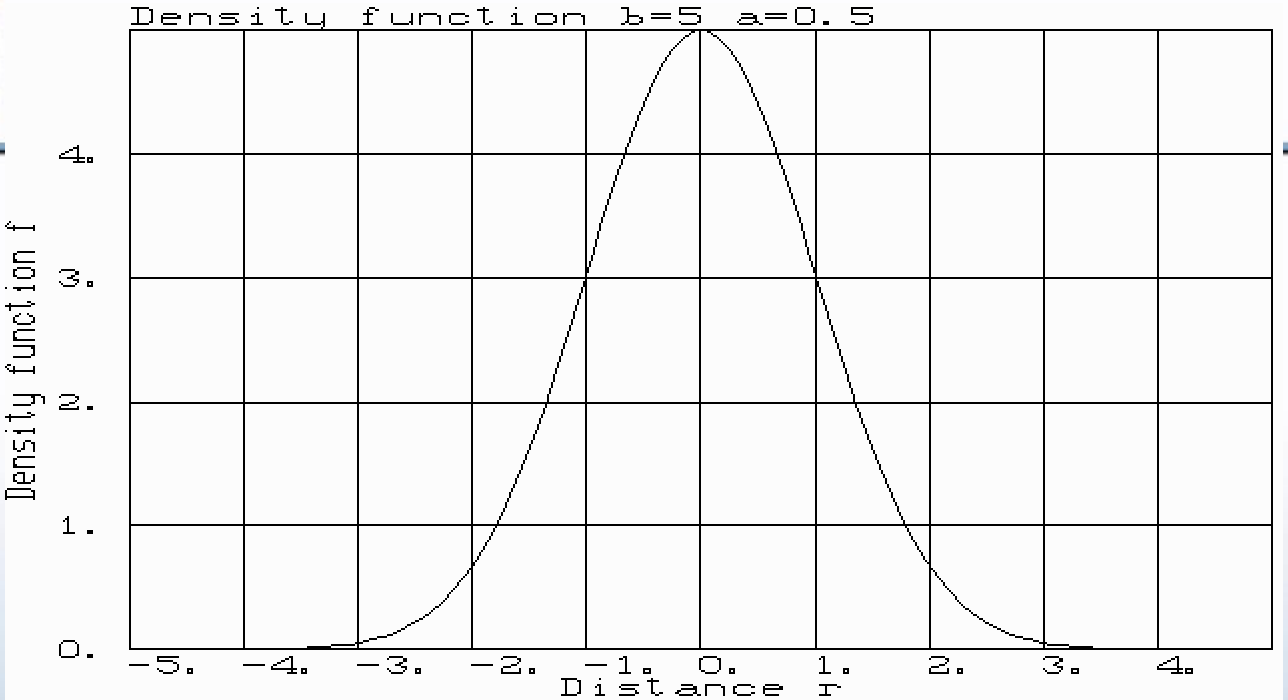
$$F_i(r_i) = b_i\, e^{-a_i r_i^2}$$

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$$

(x,y,z) are coordinates of the given point P,

(x_i,y_i,z_i) are coordinates of the *i*-th skeleton point

Blobby model



Properties:

- Exponential field does not fall to zero;
- All elements contribute to the field in any point (global influence);

Metaballs

Nishimura et al. [1985]:

$$f(r) = \begin{cases} b(1 - 3r^2 / d^2), 0 < r \le d/3 \\ 1.5b(1 - r/d)^2, d/3 < r \le d \\ 0, r > d \end{cases}$$

r is distance to the skeleton point,
d is its radius of influence

Soft objects

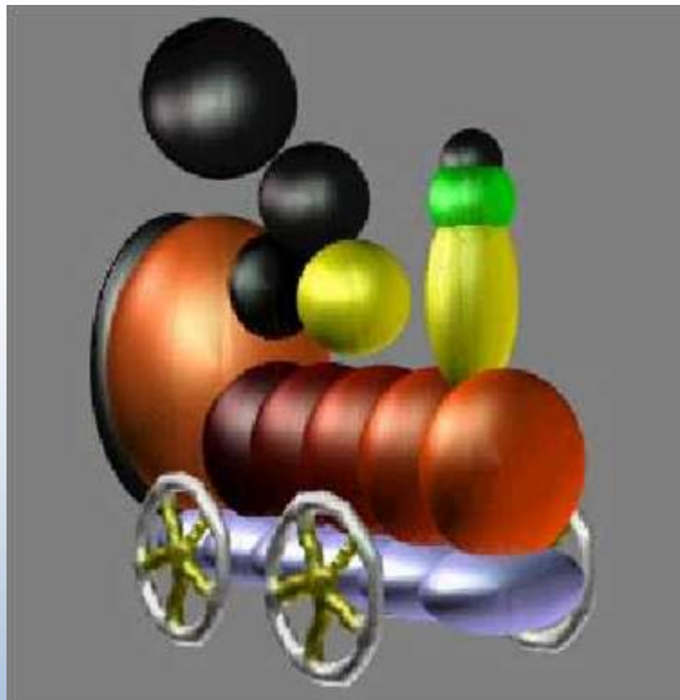
G. Wyvill et al. [1986]:

$$f(r) = \begin{cases} 1 - \frac{22r^2}{9d^2} + \frac{17r^4}{9d^4} - \frac{4r^6}{9d^6}, 0 < r \le d \\ 0, r > d \end{cases}$$

Other Skeletal Elements

- Straight line segment
- Parametric curve
- Polygon
- Parametric surface
  - Numerical distance calculation
  - Unwanted bulges

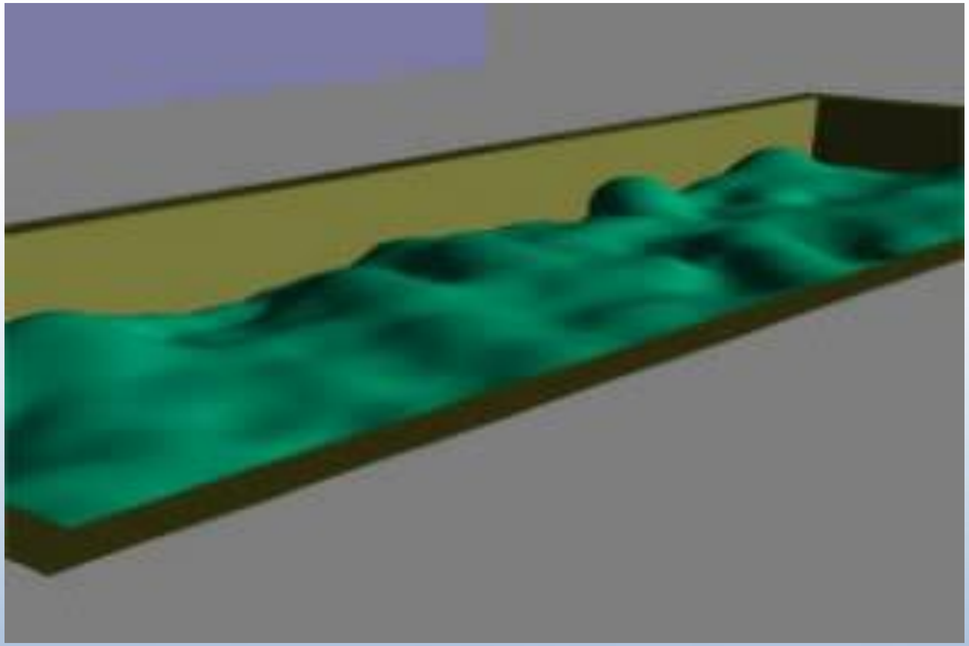
The Soft Train (1986)



$$F_{total}(P)=\sum_{i=1}^{i=n} c_i F_i(r_i)$$

Slide courtesy of Brian Wyvill

SOFT (SIGGRAPH 1986)



Courtesy of Brian Wyvill

Convolution Integral

$$f(X) = \int_{R^3} s(P)h(X - P)dP$$

- *s(X)* is a predicate function defining geometry of the skeletal element
- *h(X)* is a convolution kernel

Heavy numerical calculations

Superposition of Convolutions

$$h \otimes (s_1 + s_2) = h \otimes s_1 + h \otimes s_2$$

$\otimes$ means convolution

- $s_1 + s_2$ means union of skeletal elements

h is the kernel

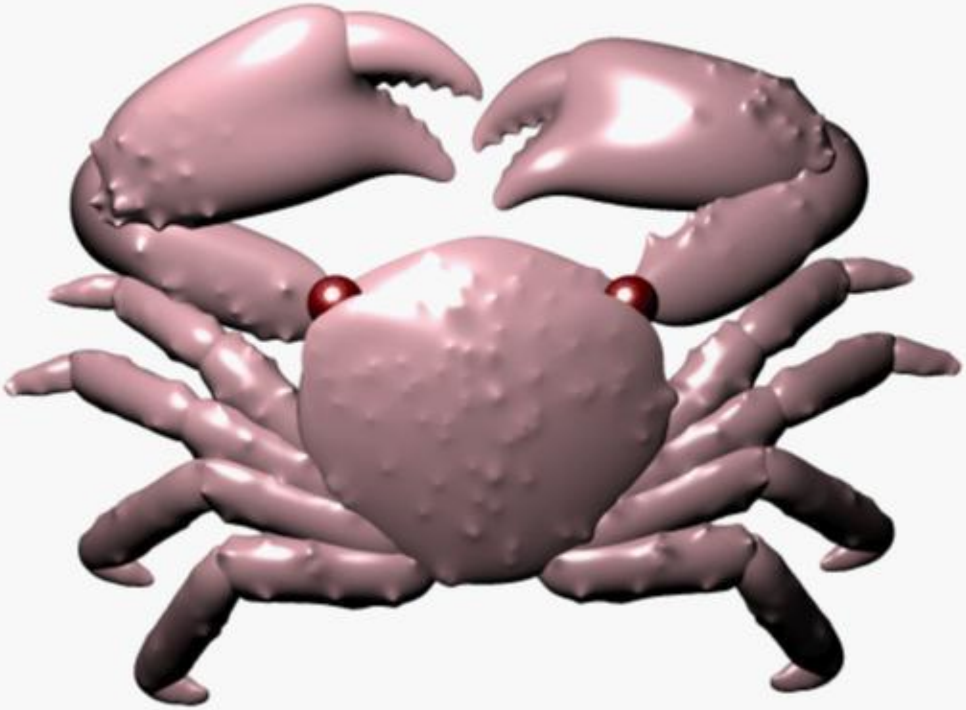
Analytical Solutions

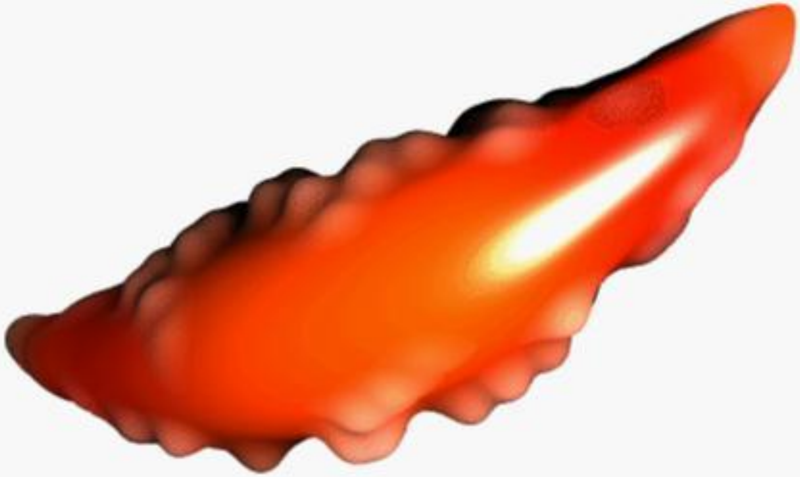
$$h(X) = 1 \big/ \left(1 + a^2 \|X\|^2\right)^2$$

Sherstyuk [1998]:

- Point
- Segment
- Circular arc
- Triangle
- Plane

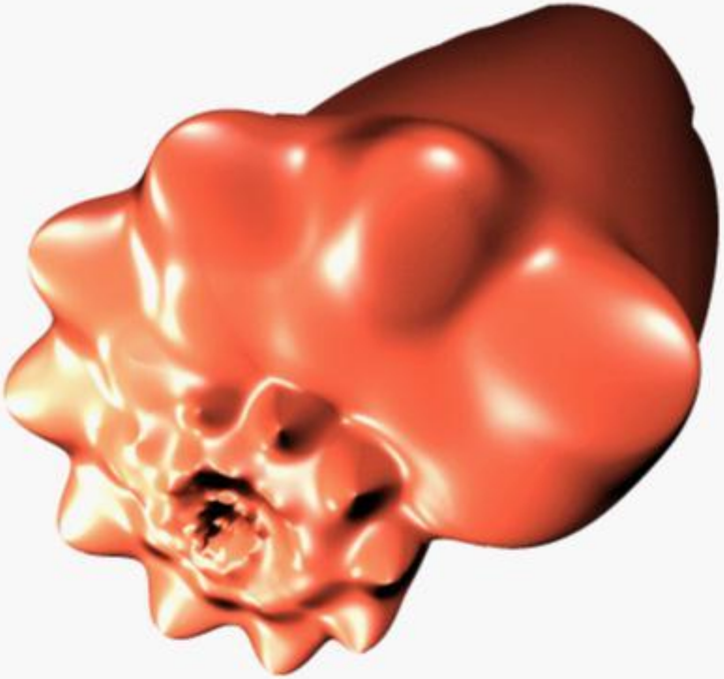
Convolution surfaces by A. Sherstyuk [SMI99]





Images courtesy of Andrey Sherstyuk

Convolution surfaces by A. Sherstyuk [SMI99]



Images courtesy of Andrey Sherstyuk

Advantages of Skeletal Models

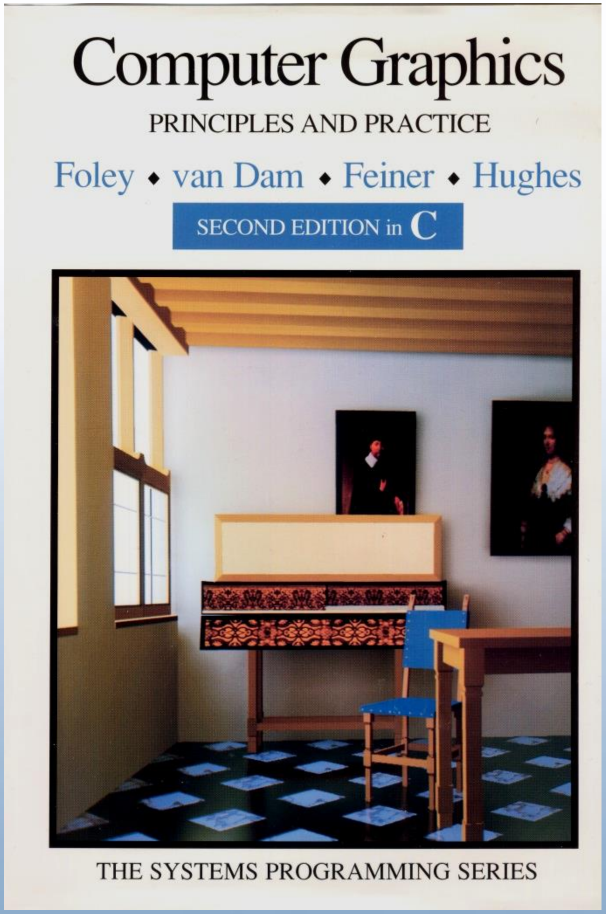
- Intuitive representations for *natural* objects (molecular shapes, liquid and melting objects, animal and human body);
- Automatic shape *blending*;
- Skeletons are easily manipulated and displayed;

Advantages of Skeletal Models

- More concise representation than parametric surfaces;
- Complex shapes can be modeled with few primitive elements;
- Easy to add new primitive types by defining a new skeleton or a new field function.

References

- James D. Foley, Andries van Dam, Steven K. Feiner, John F. Hughes, Computer Graphics: Principles and Practice (2nd Edition in C), Addison-Wesley, Reading, MA, 1997.



References

- **Introduction to Implicit Surfaces,** Ed. J. Bloomenthal, Morgan-Kaufmann, 1997.
- Bloomenthal J., Shoemake K., **Convolution surfaces**, SIGGRAPH’91, Computer Graphics, vol.25, No.4, 1991, pp.251-256.
- McCormack J., Sherstyuk A., **Creating and rendering convolution surfaces**, Computer Graphics Forum, Vol. 17, No. 2, 1998, pp. 113-120.
- Wikipedia, www.wikipedia.org
- OpenGL Performer Programmer's Guide, SGI, 2004.